

Tutorial Root Finding

A. Bisection method

1. Consider finding the root of $f(x) = x^2 - 3$. Let $\epsilon_{\text{step}} = 0.01$, $\epsilon_{\text{abs}} = 0.01$ and start with the interval $[1, 2]$.
2. Consider finding the root of $f(x) = e^{-x}(3.2 \sin(x) - 0.5 \cos(x))$ on the interval $[3, 4]$, this time with $\epsilon_{\text{step}} = 0.001$, $\epsilon_{\text{abs}} = 0.001$.
3. Find the root of $f(x) = x^6 - x - 1 = 0$ accurate to within $\epsilon = 0.001$. Given that $x_a = 1$ and $x_b = 2$.
4. Use 3 iterations of the bisection method to determine the root of $f(x) = x^3 - 7.5x^2 + 17.75x - 13.125$. Employ initial guesses of $x_a = 1.2$ and $x_b = 2$.
5. Find the root of $f(x) = x^2 - x - 2 = 0$ accurate to within $\epsilon = 0.002$. Given $x_a = 1$, $x_b = 4$.

B. Newton's method

6. Find the value of x if $x^3 = 20$ using Newton-Raphson method for three iterations.
7. $f(x) = x - 2 + \ln x$ has a root near $x = 1.5$. Use the Newton-Raphson method to obtain a better estimate.
8. The function $f(x) = x - \tan x$ has a simple root near $x = 4.5$. Use one iteration of the Newton-Raphson method to find a more accurate value for the root.
9. Estimate the root of $f(x) = 0.9x^2 + 1.7x - 5$ employing an initial guess of $x_0 = 1$ accurate to within $\epsilon = 0.001$.
10. Estimate the root of $f(x) = e^{-x} - x$ employing an initial guess of $x_0 = 0$ accurate to within $\epsilon = 0.001$.

C. Secant method

11. Find the value of x if $x^3=20$ using Secant method for three iteration, where $x_0=4$ and $x_1=5.5$.

12. Use the secant method to estimate the root of $f(x) = e^{-x} - x$. Start with initial estimates of $x-1 = 0$ and $x_0 = 1.0$.