Tutorial 13: Numerical Integration

1. Use
   a) Trapezoidal rule
   b) Simpson's $1/3$ rule
   c) Multiple-application Simpson's $1/3$ rule, with $n = 6$
   d) Simpson $3/8$ rule
   e) Multiple-application Simpson $3/8$ rule, with $n = 6$

   to evaluate the integrals

   i) $\int_0^3 (1 - e^{-x})dx$

   ii) $\int_{-2}^4 (1 - x - 4x^3 + x^5)dx$

   iii) $\int_0^{\frac{\pi}{2}} (8 + 4\sin x)dx$

2. Integrate the function using **Trapezium Rule** with step sizes $h = 0.5$ and $h = 0.25$

   $\int_{1.25}^{2.25} x^2 e^{3x} dx$

   Given the true solution $\int_{1.25}^{2.25} x^2 e^{3x} dx = \left[(1/3)x^2 e^{3x} - (2/9)xe^{3x} + (2/27)e^{3x}\right]_{1.25}^{2.25}$
   Calculate the percent relative error for problem a(i). Comment on the accuracy of your answers.

3. Consider the following table of data, find the approximation of

   $\int_{2.0}^{2.6} f(x)dx$, using **Simpson's 3/8 rule**

x	1.8	2.0	2.2	2.4	2.6	2.8	3.0
$f(x)$	6.050	7.389	9.025	11.023	13.464	16.445	20.086

4. Use

   (a) Multiple-application Simpson's $1/3$ rule with $n = 4$, and

   (b) Multiple-application Simpson $3/8$ rule with $h = \frac{\pi}{12}$ to evaluate

   $\int_0^{\frac{\pi}{2}} 4e^{-2x}dx$ correct to five decimal places.

   Given the true value is $8 - 4\sqrt{2}$. Calculate the absolute error for approximations (a) and (b) above.

5. Suppose that the current through a resistor is described by the function $i(t) = (60-t)^2 + (60-t)\sin(\sqrt{t})$ and the resistance is a function of the current, $R = 10i + 2i^{\frac{2}{3}}$. Compute the average voltage over $t = 0$ to 60 using 6 segments Simpson's $1/3$ rule.

   (Hint: $v(t) = \int i(t).R\ dt$)