Exercises on Numerical Solution to First Order Initial Value Problems

Gede Pramudya

June 1, 2016

Lesson Learning Outcome

Upon the completion of this tutorial, students are able to solve word-problems on initial value problems of first-order differential equation using Euler's, Heun's, and Classical Runge-Kutta methods.

Instructions

1. Find the mathematical modeling, which is the corresponding First-Order Initial Value Problem in the form of
$$\frac{dy}{dx} = f(x,y),\ \ y_0 = f(x_0)$$
for each of the word problems below.
2. With the step size $h = 1$ hour, use Euler's, Heun's and Classical Runge-Kutta methods to solve the problems.

Questions

1. A model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction of the population who have heard the rumor and the fraction who have not heard the rumor. It is known that the rate constant is 0.8. Then, assume a small town has 2000 peoples. At 8 AM, 80 people have heard a rumor. We would like to know the proportion of the population who heard the rumor by 12:00 noon.
2. A certain radioactive material is known to decay at a rate proportional to the amount present. If initially there is 50 milligrams of the material present and the decay constant is 0.05, find the mass of the material in 4 hours.

Formulas

- Euler's Method:
$$y_{k+1} = y_k + hf(x_k,y_k),\ \ k = 0,1,\cdots n-1.$$
- Heun's Method:
$$y_{k+1} = y_k + \frac{h}{2}[f(x_k,y_k) + f(x_k,y_k + hf(x_k,y_k))],\ \ k = 0,1,\cdots n-1.$$
- Classical Runge-Kutta Method:
$$y_{k+1} = y_k + \frac{h}{6}[v_1 + 2v_2 + 2v_3 + v_4],\ \ k = 0,1,\cdots n-1.$$
where
$$\begin{aligned} v_1 &= f(x_k,y_k) \\ v_2 &= f(x_k + \frac{h}{2}, y_k + \frac{h}{2}v_1) \\ v_3 &= f(x_k + \frac{h}{2}, y_k + \frac{h}{2}v_2) \\ v_4 &= f(x_k + h, y_k + hv_3) \end{aligned}$$